WHAT CAN WE LEARN ABOUT COGNITIVE LEARNING PROCESSES BY ASKING THE PUPILS?

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The focus is on the metacognitive awareness of ten high-achieving high school pupils from Denmark and England and their cognitive learning process. Un-structured interviews in smaller groups investigates how they say they learn a mathematical concept that is new to them. I develop the "CULTIS model for analysis" (Consciousness, Unconsciousness, Language, Tacit, Individual, and Social). These are six themes in which various psychological learning theories express themselves. I conclude that the pupils can talk about their learning using own words and I can identify fitting theoretical notions. The pupils each have their own way of learning, however, there are similarities. Seemingly contradictory theories "works" within single pupils.

INTRODUCTION

This reports a recently finished Ph.D. study (Dahl, 2002). It investigates how ten high school pupils (age 18-20) say they learn a mathematical concept that is new to them. There are four Danish pupils (*gymnasium, A-niveau*) and six English (*A-level, AS Level Mathematics*). The pupils have been taught mathematics at the highest level possible and are high-achieving. Various psychological learning theories are used to get a greater understanding of what the pupils say.

METHODOLOGY

Schoenfeld discusses the concept of metacognition, which can be understood either as knowledge about or regulation of cognition (Schoenfeld, 1992, p. 334). Knowledge about cognition means to have relatively stable information about one's own cognitive processes. This knowledge develops with age and "performance on many tasks is positively correlated with the degree of one's metaknowledge" (Schoenfeld, 1985, p. 138). Metacognition, understood as regulation of cognition, includes the planning before beginning to solve a problem and the monitoring and assessing "on-line" during problem-solving and learning (Schoenfeld, 1992, p. 355). Furthermore, good problem solvers "maintain an internal dialogue regarding the way that their solutions evolve" (Schoenfeld, 1985, p. 141). I therefore assume that high-achieving pupils have knowledge of how they learn mathematics and it will be relevant to know the "internal dialogue" as this might give a picture of the learning process.

The teachers have been asked to select the best pupils. The four Danish pupils are interviewed as one group and the six English pupils are interviewed in pairs. Each session

with the English pupils consists of three phases, but the Danish pupils do not meet the second phase:

1. Semi-structured group interview. The pupils are asked to describe a usual mathematics lesson, what they *do* when they meet some new mathematics, how they know if they have learnt it, and they have been asked to describe their learning process. This lasts about 10 minutes.

2. Participant observation. They get a sheet (Nelson & Wilson, 1990) with some basic knot theory they are supposed to get *some* understanding of. They have the opportunity to involve the observer in the learning process, but they do not. They also get a sheet with questions to make them think about what they do while working with the mathematics. This lasts about 15 minutes.

3. Unstructured qualitative group interview with open-ended questions. They are asked what they *did* in order to learn, why they did the things observed, how and why it helped, what they would *do next* to fully understand this mathematical concept, if (how) this learning situation is different from the usual, and how they would present this mathematics to the class. The method is to listen and probe to "open-up" in-talk and reformulate or challenge their answers to get a more elaborate explanation. The attempt is to try not to ask leading questions and to keep an open mind. Periods of silence are allowed to not interrupt anyone's chain of thoughts. The pupils give examples from either the knot theory or some other mathematics they have met. This lasts about 30 minutes. The style of the interview is chosen to avoid a self-fulfilling process if both interview and model for analysis are strongly influenced by theory.

THE PSYCHOLOGY OF LEARNING MATHEMATICS

To have a range of possible explanations of what the pupils tell, mainly the following theorists are used: Glasersfeld (1995), Hadamard (1945), Krutetskii (1976), Mason (1985), Piaget (1962, 1970), Polya (1971), Sfard (1991), Skemp (1993), and Vygotsky (1962, 1978). To some extent they have a similar focus as this study and they are "classics". Below I explore in which themes these theories express themselves and I develop the so-called CULTIS model for analysis. The pupils' explanations are thus classified into these six different themes. The six themes are also divided into three binary opposite-pair. The fact that there is, for instance, a theme named 'social' means that when I read through the transcribed interviews, I label some of their remarks 'social' without, at first, going any deeper into what the pupil means. The themes are thus overall boxes to sort various topics the pupils mention. The themes "cut" the theories into "modules". The themes are: Consciousness-Unconsciousness; Language-Tacit; Individual-Social (CULTIS). To some extent the themes interact and overlap each other but each have their own identity.

First pair of themes; Theme 1: Consciousness

According to Polya (1971) and Mason (1985), working with mathematics has three phases. First: 'enter' the problem, understand the problem and device a plan. Second: carry out the plan, and the third is a revision of the whole process.

Polya states that a good idea of a plan is "based on past experience and formerly acquired knowledge. Mere remembering is not enough for a good idea, but we cannot have any good idea without recollecting some pertinent facts" (Polya, 1971, p. 9). The pupil must

furthermore understand the problem before starting to work on it, and the pupil should "desire its solution" (Polya, 1971, p. 6), or in other words, be motivated. In Activity Theory motivation plays the major role (Mellin-Olsen, 1989, p. 16-17), but is in this paper mentioned as one item of many. According to Polya, it is a practical skill to be able to solve problems and since we require all practical skills by imitation and practice, this also applies for solving mathematical problems (Polya, 1971, p. 4-5). Also Sfard (1991, p. 18) states that operational understanding is the only way to 'get in touch' with abstract constructs. Mason writes that practice is important but without reflection it may leave no permanent mark, and that it also needs time. Mason also states that to support mathematically thinking one needs a questioning, challenging, and reflective atmosphere (Mason, 1985, p. 153).

First pair of themes; Theme 2: Unconsciousness

Hadamard (1945, p. 56) states that there are four stages in learning: preparation, incubation, illumination, and verification. Conscious work is preparatory to the illuminations. Polya states that "only such problems come back improved whose solution we passionately desire ... conscious effort and tension seem to be necessary to set the subconscious work going" (Polya, 1971, p. 198). The illumination is generally preceded by an incubation stage where the solving of the problem is completely interrupted (Hadamard, 1945, p. 16). The first stage in solving a problem is therefore to work in a very concentrated manner on it. What is experienced as sudden inspiration "is the result of previous protracted thinking, of previous acquired experience, skills, and knowledge" (Krutetskii, 1976, p. 305).

Second pair of themes; Theme 3: Language

Different theorists discuss the indispensable role of language, words, and concept formation in learning. Vygotsky describes language as the logical and analytical thinking-tool (Vygotsky, 1962, p. viii) and that thoughts are not just merely expressed in words but come into existence through the words (Vygotsky, 1962, p. 125). Furthermore, as mathematics in itself is a language (Pimm, 1990, p. 2; Dahl, 1996), it becomes important also to discuss concept formations. In relation to the learning of mathematical concepts, a basic principle is that all concepts, except the primary ones, are derived from other concepts and they take part in the formation of other concepts (Skemp, 1993, p. 35). This conceptual structure is called a schema, and a schema is therefore a tool for learning as it integrates existing knowledge (Skemp, 1993, p. 37). Similarly, Tall (1991, p. 9) refers to the notions of *assimilation,* a process by which an individual adopts new information and *accommodation,* which signifies that the individual's cognitive structure must be changed. Thus, it seems that language is not essential for the creation of the basic concepts. But higher order concepts build on the basic concepts and to learn the higher order concepts, other concepts are necessary. Furthermore (Skemp, 1993, p. 29-30), an integrated conceptual structure is easier to remember than unconnected rules.

Second pair of themes; Theme 4: Tacit

There are also more negative views of language as a tool for learning. To Hadamard, "thoughts die the moment they are embodied by word" (1945, p. 75), but, however,

"signs are necessary support of thought" (Hadamard, 1945, p. 96). Piaget (1970, p. 18-19) states that "This, in fact, is our hypothesis: that the roots of logical thought are not to be found in language alone, even though language coordinations are important, but are to be found more generally in the coordination of actions, which are the basis of reflective abstraction". Thus, actions are the root of logical and mathematical thought. In relation to tacit knowledge, one can observe that a person has a certain kind of knowledge, but "on questioning, it appeared that he did not know he was doing this. Here the subject got to know a practical operation, but could not tell how he worked it" (Polanyi, 1967, p. 8). The "negative" arguments are thus centred on the general uselessness of words in thinking and learning, that language merely "supports thinking", as well as the lack of ability to describe what one is doing.

Third pair of themes; Theme 5: Individual

The individual perspective of learning is represented by for instance Glasersfeld whose epistemology is that "knowledge, no matter how it is defined, is in the heads of persons, and that the thinking subject has no alternative but to construct what he or she knows on the basis of his or her own experience. … all kinds of experience are essentially subjective (Glasersfeld, 1995, p. 1). Piaget asks what mathematical-logical knowledge is abstracted from and he finds that the basis of abstraction comes from the action itself, not the object (Piaget, 1970, p. 16-18). The individual who is learning is therefore active and the acknowledgement comes as the individual manipulates with the objects and reflects on this manipulation. Piaget talks here about reflective abstraction, which among other things means the transposition from one level of a hierarchy to another, and it means the mental process where a reorganisation of thoughts takes place. Reflective abstraction is based on coordinated actions, not individual. This therefore means that (1) language is not the main thinking-tool, (2) both individual actions and coordinated ones are performed by the individual and they both lead to abstraction, but it is the latter that leads to reflective abstractions and then to logical-mathematical knowledge. Piaget therefore finds that logical-mathematical abilities do not arise from language or linguistic competency, but from the ability to coordinate actions and operate with objects.

Third pair of themes; Theme 6: Social

In this theme, social interaction plays a fundamental role in shaping pupils' internal cognitive structure (Schoenfeld, 1985, p. 141). This process has two levels: "first between people … and then inside the child ... All higher functions originate as actual relations between human individual" (Vygotsky, 1978, p. 56-57). This process of internalisation is gradual. In the beginning a teacher controls and guides the pupil's activity, but later they begin to share the problem-solving functions, and here it is the pupil who takes the initiative while the teacher corrects and guides. At last, the pupil is in control and the teacher's role is mainly supportive. According to Vygotsky, the potential for learning is furthermore limited to the "zone of proximal development (ZPD)" (Vygotsky, 1978, p. 86). ZPD is the area between the tasks a pupil can do without assistance, and those, which require help. It is therefore essential that pupils are active and have the opportunity to be guided by a knowledgeable person. Verbal thinking is an example of a social activity. When the pupil speaks aloud, the "audible speech brings ideas into

consciousness more clearly and fully than does sub-vocal speech" (Skemp, 1993, p. 91-92). Vision is therefore individual, while hearing is collective (Skemp, 1993, p. 104).

A possible synthesis of different psychological theories?

Above is seen that the various theories are sometimes contradictory. The dualism is mainly in Theme 4 to 6. Two representatives of this are Piaget and Vygotsky, particularly about the role of the individual and the social and of language. According to Lerman (1996, p. 133), "Vygotsky's and Piaget's programs have fundamentally different orientations, and that the assumption of complementarity leads to incoherence". However, Piaget and Vygotsky have a mutual sympathetic, yet critical, view of each other (Piaget, 1962; Vygotsky, 1962). I therefore discuss the possibility of a synthesis between them as well as the possibility of a grand-theory (Dahl, 2002). The conclusion was, for now, to settle with Bohr's Principle of Complementarity, but here talk about what I express as *odd complementarity* denoting that neither psychological theory is complete, but they might not be equally *dis*-complete.

SAMPLE DATA, DISCUSSION, AND CONCLUSION

The pupils' statement can mainly fit the CULTIS model. To a certain extent they mention the same things, but there seems to be a difference between them on areas such as the role of practice, language, and individual-social issues. Below is an example from the English interviews with Pupil D and E. The essential is underlined, and in the column to the left is noted which theme the remarks have been identified to belong to. 'I' is the interviewer and 'Q' means question.

Q	I:	So, what could they have done, let's say, if they wanted to describe knot theory.
6	D:	I think this is, <u>this is the kind of thing where it is very difficult to talk about in a book</u> and to represent 3-dimensional object within a 2-dimensional way, and <u>it's where it would help to have a teacher explaining something and say pointing all this is the vertex and this is an edge</u> (E: (inaudible)) (inaudible)
	I:	Pointing at, I mean
6	D:	Yea, <u>drawing it</u> or tiny little knots <u>and say this is</u>
5	E:	<u>Depends who they are targeting it at</u> (1 sec silence), don't be so (1 sec silence) so
4		stuck up (inaudible) (laughs) and so you get to a, I don't know, <u>don't use such big words, they are aiming to people who don't understand it</u> (I: mmm, well it's part of er) and use basic, yea (I: so I mean) no, I know, but I it would seem a bit sort of (1
4		sec silence) if it if it was in a GCSE and A-level course (D: mmm) all this would have, language, <u>it wouldn't be right, it's the the way they approach it, the language, it's just too, people would struggling with the language when they are suppose to be learning the maths</u>.
Q	I:	So is there a diff, I mean, er, so maths has nothing to do with the language? Or, can you learn maths without language.
4	D:	<u>Yea</u>.
3	E:	<u>No, but you can use different language, simple language to convey a point</u>.

4 D: Cause the maths in it is quite easy, I think, well, it's not (E: I'm sure it is (giggles)) (laughs) What do you mean it is nothing really difficult what it is saying is this is what a knot is, this is (E: Yea) what a link is, and, OK, that really really simplistic, it takes a long time (laughs (inaudible)) it took me a long time to work out what they were trying (E: Yea what they were explaining) whereas the fact as soon as I, kind of translated it, I thought oh well, that's what a knot is, find that's easy.

Q I: What did you translated it (inaudible)

4 D: Into simple language (laughs) er, it er (inaudible)

Q I: You translate it before you understand it, er, so (D: (inaudible)) if you have understand, then, it, you don't need to translate it.

6 E: I think it here would be easier if the author translated (D: Yea (laughs)) rather than er leaving the reader to er (1 sec silence) to do it, I mean.

4 D: You have you have do the two together, you have to translate while you're trying to understand

The theoretical understanding of Pupil D and E, as well as the eight others, in terms of the Theme 3 and 4 is thus that some pupils say that language is the main thinking-tool, others that it hampers thinking, others that language seems to have a dual nature as it both facilitates learning and hampers learning and this does also depend on the type of language. For Theme 1, almost all the pupils talk about motivation, but there are various views of its nature. The cognitive drive is mentioned as important. A more "outer" motivation is to be forced. An "inner" motivation comes from being confused by something. Some pupils also explain that lack of motivation can be caused by lack of self-confidence. There is also a "show-off" effect. All the pupils talk about doing exercises as important for the learning process, but there are some variations. In Theme 2, most pupils talked about Hadamard's (1945) three phases. About Theme 5 and 6, it seems that most of the pupils argue that learning has both a social and an individual side. The value of the social side is mainly when one experiences problems trying to learn by self-study. After input from others, one can move on alone. This is the case for eight of the pupils. The two others emphasise the social side. Theme 5 and 6 thus complement each other "odd".

There are some utterances that did not fit into the themes. Half the pupils mention that how one is used to learn/being taught, influence on how one learns later on. For instance: Pupil Z explains that how they learn is influenced by the fact that they have been trained to have a visual cognition, and therefore they learn most things through their eyes. Pupil A says that the learning strategies one knows and uses, are connected with the ways one has been taught to do things. Pupil D explains that it is hard to adapt to a different teaching style. Thus, the teaching methods must be part of, what I would express as a *zone of proximal teaching (ZPT)*, inspired by Vygotsky's ZPD. Similarly, one could here state that if a (new) teacher (perhaps a new school) uses teaching methods that are too "far away" from the teaching styles the pupils are used to, the pupils might not learn. Furthermore this leads to a conclusion that a change of teaching styles ought to be gradual. A pupil's previous experience of learning, his learning history, does therefore to

some extent influence how he later on is able to learn. This means that the single teacher's method of teaching is a factor that has consequences for the pupil's later learning successes. As this study confirms that some pupils are able to talk about their learning processes, one might argue that a discussion and greater awareness of this between pupil and teacher might, improve the learning.

Another further result is that different learning theories seem to fit different branches of mathematics and the types established go across country and gender.

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